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Controlled quantum state transfer in *XX* spin chains at the Quantum Speed Limit

D.S. Acosta Coden [a], S.S. Gómez [a], A. Ferrón [a], O. Osenda [b]

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Highlights

- We achieve control of the dynamics of a spin chain at times near the QSL with one or two actuators.
- The control schemes are robust against static disorder achieving near perfect state transfer.
- Optimal Control Theory provides simple, reliable and efficient pulses to control excitations transfer in spin chains.

Abstract

The Quantum Speed Limit (QSL) can be found in many different situations, in particular in the propagation of information through quantum spin chains. In homogeneous chains it implies that taking information from one extreme of the chain to the other will take a time $O(N/2)$, where N is the chain length. Using Optimal Control Theory we design control pulses that achieve near perfect population transfer between the extremes of the chain at times on the order of $N/2$. Our results show that the control pulses that govern the dynamical behavior of chains with different lengths are closely related. The pulses were constructed for control schemes involving one or two actuators in chains with exchange couplings without static disorder. Our results also show that the two actuator scheme is considerably more robust against the presence of static disorder than the scheme that uses just a single one.

Introduction

The transfer, or transmission, of quantum states in different communication channels has been touted as a task of paramount importance to routinely implement quantum information processing (QIP) and technologies. Since the paper of Sougato Bose in 2003 [1] dealing with the transfer of states in quantum spin chains the area has grown enormously and the understanding of the field has been collected in books [2] and other excellent reviews [3]. Bose [1], introduced the simplest transfer protocol and the averaged fidelity over all the possible initial pure states, that are to be transferred from one extreme of a spin chain to the other, as the figure of merit that quantifies the overall performance of the spin chains as a communication channel. Also from the beginning, it was understood that the severe restrictions imposed by the fragility of the quantum states to be transferred required *perfect* or near perfect quantum transmission so the errors do not spoil the state for further use after it has been transmitted.

The transmission channel is constructed from copies of a given physical system. The copies are spatially distributed along a line or in other spatial distributions [4] and might or might not interact with a surrounding environment. Many different physical systems have been considered as the basic unit of the communication channel, quantum dots [5], superconductor qubits [6], nuclear spins [7], cold atoms or ions trapped in arrays [8], [9] or coupled waveguides [10]. Anyway, most theoretical studies focus on Heisenberg or *XY* spin chain models since these models capture the basic physical traits that are of interest. In the case of quantum dots chains most theoretical studies deal with the Hubbard model [11] although other approaches are feasible [12].

In models without environment, *i.e.* with unitary dynamics, there are two main approaches to achieve perfect or near perfect state transfer. In the simpler approach the Hamiltonian unitary dynamics takes the state to be transferred from one extreme of the spin chain to the other and the spin chain Hamiltonian is time-independent. Several possibilities have been studied, for example, chains that achieve perfect transmission by engineering all the spin couplings to special values [13], [14], [15], conclusive and perfect transmission using to parallel spin chains [16], [17], boundary controlled spin chains [18], [19], [20], optimal dynamics [21], [22], and so on. The second approach assumes that the system will be driven by *control* fields, so the dynamical behavior is governed by the free Hamiltonian and a number of external fields. Both approaches share some features, usually the dynamics is confined to some sub-space of the whole Hilbert space, most commonly a sub-space with fixed number of excitations or magnetization. This assumption reduces drastically the size of the computational problem and makes feasible to study systems with static disorder.

Quantum spin chains of different kinds are *controllable* in the whole Hilbert space [23], [24], [25] and in sub-spaces of fixed magnetization [26] even with very few actuators, moreover Heisenberg spin chains are controllable with just a single actuator. Controllable in the whole Hilbert space means that, using an adequate forcing, it is possible to go from an arbitrary initial state to whichever target state is chosen. There are numerous papers dealing with the controllability of these models using an external magnetic field applied to the first spin, *i.e.* the spin where the initial state is prepared to be transmitted. Actually, besides the transmission of quantum states there are other quantum entangling gates that can be controlled along a spin chain [27]. The preparation of particular target states has also been considered [28], [29], as well as the possibility of controlling the state transfer applying fields to all the sites [30], [32].

The logic behind a reduced number of control fields, and that they are applied to the extremes of the chain, is based on what can be feasible on actual implementations, where each control field could add new sources of errors and the application of an external field to a single element of a truly nano scale device is troublesome at least, in particular it is located in the middle of other elements alike.

Inspired by control techniques typical of Nuclear Magnetic Resonance, the design of control pulses based on trains of square pulses of varying length in time has been intensely studied in Heisenberg spin chains [42], [43]. In these studies the strength of the magnetic field applied to the first site of the chain is constant during an interval and different optimization procedures are used to determine the length of the interval and the strength of the field. As is common in studies involving Optimal Control Theory (OCT) the control time is fixed *a priori*, in the case of a single control field this time has been chosen on the order of N^2 [26], [27], [31], where N is the number of spins in the chain, but for more involved control algorithms shorter times, compatible with QSL, can be achieved [32]. It is worth to mention that using two control fields and OCT Zhang et al. [33] have found that the XX chain achieves state transfer with high fidelity for times that depend linearly with N.

The works of Bose and co-workers using time-dependent exchange couplings are closely related to the present work. Using a single actuator and OCT [34] and “ramping” the odd exchange couplings [35] it is shown that Heisenberg models of quantum dot arrays achieve quantum state transfer with a very good quality at times on the order of N^2.

The QSL refers to the theoretical maximum speed at which quantum information can be transferred through a quantum system. It appears in different contexts and physical systems, P. Pfeifer [36] studied how fast a given quantum state can change in time and found lower and upper bounds for the decay and transitions of quantum states, evolving under a time-dependent Hamiltonian, in terms of the energy uncertainty of the initial and final state. The fact that quantum computation and quantum communication performances, in particular the speed at which they can be operated, is strongly limited by the physical traits of the systems where they are implemented was early recognized and analyzed by S- Lloyd in Reference [37], while its appearance in quantum state transfer in quantum spin chains have been intensely studied [15], [32], [38]. In this context, the QSL is the fastest speed (or shorter time) at which a state prepared at one extreme of the chain can be retrieved at the other extreme of the chain with a very high fidelity. In Reference [39] it is shown that the minimum transfer time for homogeneous spin chains with a exchange coupling J is equal to $N/2J$ and $N/4J$ for chains with Ising or Heisenberg interactions, respectively.

In this paper we present strategies to control state transfer in *XX* spin chains that achieve fast transfer times, $O(N/2)$, with very high fidelities and with smooth pulses that control the coupling between the first pair, or the first and last, pair of spins. We were motivated to study the design of pulses using OCT to modulate the spin couplings by the work of Kandel et al. [40] that consider spin-state transfer via Heisenberg exchange modulating in a quantum dot chain. Our results show that the shape of the pulses for different chain lengths is closely related, and that the external forcing also shows the effect of the QSL, restricting the time interval in which it is necessary to achieve the transfer.

The paper is organized as follows. In sec. 2, we present the model Hamiltonian for the spin chain with and without static disorder and we analyze how to design pulses in order to transfer excitations from the first site to the last one. In sec. 3 we address the possibility of controlling excitations using the optimal control theory in order to design a one actuator control strategy. Sec. 4 deals with the possibility of performing control tasks with a slightly different strategy, in this case we use pulses designed for two actuators. In both sections we analyze the inclusion of static disorder and the effects in the control strategies. Finally, in Sec. 5 we summarize the conclusions of our work.

Section snippets

Model Hamiltonian, with and without disorder and dynamics

We consider a spin-1/2 chain with interactions between nearest neighbors, described by the Hamiltonian $H_{XX} = -\frac{1}{2}\sum_{i=1}^{N-1} J_i\left(\hat{\sigma}_i^x\hat{\sigma}_{i+1}^x + \hat{\sigma}_i^y\hat{\sigma}_{i+1}^y\right)$ where σ_i are the Pauli matrices, N is the chain length and J_i are the exchange interaction couplings. In this work we set $J_i = 1$ for $i = 2, 3, \ldots, N-2$ and $J_1 = J_{N-1}$ different from the rest of the couplings. The Hamiltonian in Eq. (1) has been studied extensively in the context of quantum state transfer, see Reference [2] and References therein. It is exactly…

Control of spin excitation in a *XX* chain:one actuator

In this section we present results corresponding to the one actuator OCT protocol, as defined in Eq. (13) using $h_R = 0$. The main objective of the present protocol is to drive an excitation from the first site to the last one through the manipulation of the coupling between the first and second site.…

Control with two actuators

This Section is dedicated to analyze a different control strategy, using two actuators instead of just a single one, and assess if the physical traits found in the previous Section with respect to the control pulses, control time and robustness against static disorder are shared by more complex control strategies. To this end we implemented control equations where the first and last exchange couplings are time-dependent control functions. Despite the excellent controllability and the high…

Conclusions

The results presented in the previous Sections show that, using OCT, it is possible to design smooth, simple and reliable control pulses, that transfer population from one extreme of a *XX* spin chain to the other. The transfer can be achieved choosing the arrival time, as long as this time is longer or similar to the time compatible with the QSL, $T_{QSL} \approx N/2$. Moreover, in the one-actuator scheme, if the arrival time is chosen similar to T_{QSL} the pulses will have very good properties as low fluence…

CRediT authorship contribution statement

D.S. Acosta Coden: Conceptualization, Methodology, Software, Writing – review & editing. **S.S. Gómez:** Conceptualization, Methodology, Software, Writing – review & editing. **A. Ferrón:** Conceptualization, Methodology, Software, Writing – review & editing. **O. Osenda:** Conceptualization, Investigation, Methodology, Writing – original draft, Writing – review & editing.…

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.…

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